Rotor solidity effects on the performance of vertical-axis wind turbines at high Reynolds numbers

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Abstract.

Vertical Axis Wind Turbines have yet to see wide-spread use as a means of harvesting the kinetic energy of the wind. This may be due in part to the difficulty in modeling the relatively complex flow field and hence performance of these units. Additionally, similar to Horizontal Axis Wind Turbines, VAWTs are difficult to properly test in a conventional wind tunnel. Typically Reynolds numbers cannot be matched or the turbine geometry must be altered, limiting the applicability of the results. Presented in the following is a set of experiments in a specialized, high-pressure wind tunnel used to achieve high Reynolds numbers with a small-scale model. The performance change of this model is investigated at two different solidities and over nearly a decade of Reynolds numbers (based on diameter: $600,000 \leq Re_D \leq 5 \times 10^6$). The non-dimensional power coefficient displays behavior consistent with Reynolds number invariance, regardless of solidity. In addition, the change in performance as this limit is approached also shows no direct dependence on the solidity for the VAWT geometry used in this study. The results of this work have direct application for modeling and simulation efforts concerning the performance of new VAWT designs.

1. Introduction

Despite continued research effort over the past several decades, vertical axis wind turbines (VAWTs) have yet to see wide-spread acceptance in the commercial wind industry. The flow field created by a VAWT has proven to be much more difficult to model than that of a similarly sized horizontal axis wind turbine (HAWT). This difficulty arises from the highly three-dimensional flow field created by each turbine blade and the turbulent, asymmetric wake shed behind the units. However, from an experimental and computational point of view, the governing non-dimensional parameters for both HAWTs and VAWTs remain the same:

$$Re = \frac{U^* L^*}{\nu}; \qquad \lambda = \frac{\omega R}{U^*}; \qquad Ma = \frac{\omega R}{a}; \tag{1}$$

where ω is the angular velocity of the rotor, R the rotor radius, ν is the kinematic viscosity of the working fluid, and a the speed of sound. The characteristic velocity scale is represented by U^* (such as the free-stream velocity) while L^* is the characteristic length scale (such as the rotor diameter or chord length). Both Re and λ are equally as difficult to match in a typical laboratory-scale atmospheric wind tunnel for a VAWT as for a HAWT owing to the inverse scaling relationship with velocity. The only free parameter available for matching Re in an atmospheric wind tunnel is the tunnel velocity, U, appearing in the numerator of Re, but the denominator of λ. For a typical length-scale reduction of $R_{model}/R_{full-scale} \approx 1/100$ the rotation rate of the model, ω, must be increased 10,000 times for Re and λ to match. Rotational rates of this magnitude are not only mechanically infeasible, but the resulting tip velocities make it impossible to match the Mach number, Ma.

Despite these difficulties, the scale effect on VAWT performance remains a subject of interest, especially at full-scale Reynolds numbers. Due to the increased cost and complexity of performing large-scale experiments or full numerical simulations, only a few studies at or near full-scale Re values have been accomplished. Furthermore, if the effect of changing solidity was also studied, the available results are even more sparse. Here solidity is defined as:

$$\sigma = \frac{nc}{2R} \tag{2}$$

where c is the blade chord and n the number of blades. Early work suggested that a value of $\sigma = 0.3$ would return the highest turbine efficiency [1]. Although commercial field turbines can often be higher solidity, with σ values near or exceeding unity [2], warranting their inclusion in research work. A final note is that different model geometries are often used making direct comparisons difficult (see [3] for a review of VAWT configurations).

Numerical simulations performed on the H-rotor VAWT using various solidities have observed high Reynolds number trends [4]. For each solidity tested ($\sigma = 0.13, 0.25, 0.47, 0.79$), the turbine efficiency increased with Reynolds number across all tip speed ratios. Furthermore, although the specific value of the maximum efficiency did depend on σ, all cases displayed asymptotic behavior as Reynolds number was increased.

Several laboratory and field measurements of turbine efficiency have been performed on large or full-scale models [5, 6, 7]. In all cases the general trends compare well with the H-rotor simulations: model performance was increased for all tip speeds and all solidities ($\sigma = 0.13$ to 0.3) by increasing the Reynolds number. However, no clear plateau behavior was observed for the two smaller-scale experiments (one wind tunnel and one field test) [5, 6]. It was not until field experiments using a much larger turbine were performed [7] was any plateau-like behavior observed in the performance, and only at a relatively low $\sigma = 0.14$. This indicates that a very large Reynolds number is potentially required for scale-independent behavior.

Present work aims to further the understanding of Reynolds number and solidity changes on VAWT performance. To accomplish this, the typical model scaling issue for kinetic energy harvesting devices such as HAWTs and VAWTs was bypassed by using highly compressed air as the working fluid [8, 2]. This allows for a reduction in the kinematic viscosity by over two orders of magnitude, which facilitates matching the tip speed ratios and high Reynolds numbers of the field-scale units simultaneously. To accomplish this, a specialized, high static pressure wind tunnel known as the High Reynolds number Test Facility (HRTF), has been utilized along with small-scale wind turbine models designed specifically to operate in the HRTF. In the following work, we investigate changes in VAWT performance for two different solidities ($\sigma = 0.67$ and 1.12) at high Reynolds numbers. This work has direct impact on modeling and simulations if it is desired to match the high-Reynolds number performance of field turbines.

2. Experiment Description

The experimental facility used for this study, known as the HRTF, is a recirculating-type, high-static pressure, low-velocity wind tunnel which employs compressed air as the working fluid. The facility is designed to support static pressures up to 233 bar and free-stream velocities of 10 m/s. This facility has been used in previous work on horizontal and vertical axis wind turbines [8, 2], as well as two-dimensional airfoil tests [9], zero-pressure-gradient turbulent boundary layers [10], and high-Reynolds number studies in the wake of a suboff model [11, 12]. Further details regarding this facility can be found in those studies.

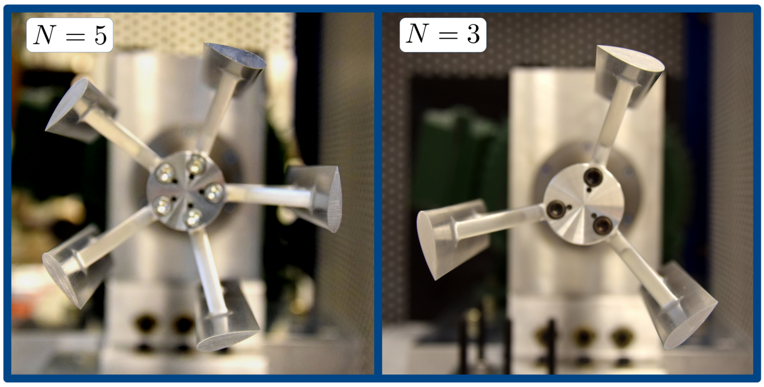

Figure 1. Top view image of the 5 blade and 3 blade VAWT models.

The current experimental configuration utilized the geometry of a commercially-available wind turbine design from Wing Power Energy (WPE) Inc. A 1:22.5 scale model was constructed specifically for use in the HRTF. Recent work on this project focused on the scaling behavior of the power coefficient with Reynolds number as compared to the field measurements [2]. Additional information regarding the specific rotor geometry used can be found in that work. The current set of experiments focus on investigating the effect of solidity using the same base geometry for the turbine. This was accomplished by reducing the number of blades on the model from five to three via the use of a different mounting hub as shown in figure 1. The configurations studied had corresponding solidities of $\sigma = 0.67$ for the 3 blade, and $\sigma = 1.12$ for the 5 blade. All geometric details between the two models remained the same except two blades and their support struts were removed for the 3 blade model. This allowed for a direct comparison between the two units at matched Reynolds numbers.

Model experiments in the HRTF were performed at fixed Reynolds number based on diameter, Re_D. Typically, tunnel static pressure was set and a mean velocity chosen which corresponded to the desired Re_D. The turbine was then allowed to self-start, completely unassisted until it reached a steady-state, unloaded, rotational speed. The braking load on the turbine was then altered to control ω and hence produce various λ values. The reported power data can be considered the true, aerodynamic power as mechanical losses in the setup were estimated to be of the same order as the measurement uncertainty. Preliminary bench testing external to the HRTF indicated excessive mechanical vibration of the 3-bladed model at specific rotational speeds. During experiments in the HRTF, these rotation rates were avoided to reduce any experimental error.

3. Results

The main point of comparison for these experiments is the power coefficient. It is defined to be the mechanical power, which is the product of the measured shaft output torque and rotation rate, appropriately non-dimensionalized by flow and rotor geometric parameters as

$$C_p = \frac{\tau\omega}{\frac{1}{2}\rho U^3 A} \tag{3}$$

where $A = S \times D$ is the area swept by the rotor. The global performance changes associated with altering rotor solidity across a range of different Reynolds numbers is examined in this section. Then the trends observed when approaching the high Reynolds number limit are discussed.

3.1. Power Coefficient

Figures 2 (a) and (b) show the power coefficient as a function of the tip speed ratio. Each power curve was measured at a fixed value of Re_D, with the line color corresponding to the magnitude of the average Re_D for each run. To ease comparisons in regions where data is sparse, 3^{rd} order polynomials have been fitted to the experimental points as visual guides only and are shown as solid lines. Immediately evident from these two plots is the performance advantage of the 3 blade, lower solidity unit. In some cases an increase of over 16% was seen in the peak power coefficient at comparable Re_D. Also evident is the shift in operational λ values. Note that the turbines were completely driven by the incoming flow, and the measured power is the true mechanical power extracted from the flow by the turbine. The 3 bladed unit operated at higher values of λ than the 5 blade turbine, with the maximum power coefficient typically residing at $\lambda \approx 1.3$ instead of $\lambda \approx 1$ for the 5 blade.

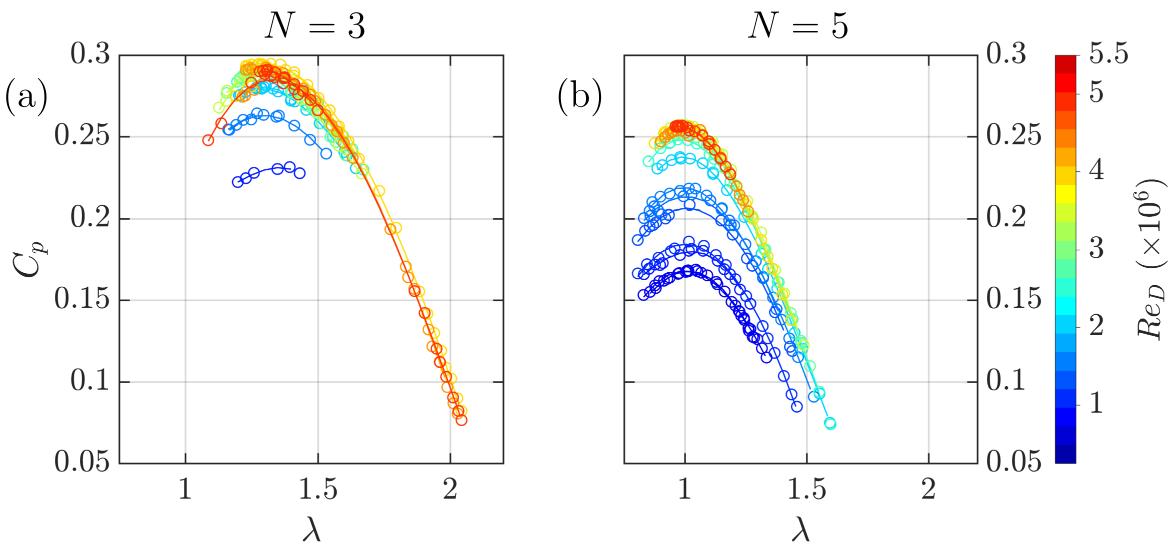

Figure 2. Power coefficient trends with tip speed ratio and Reynolds number based on diameter for the 3 blade model in (a), and the 5 blade model in (b).

Despite these differences, figure 2 shows that both turbine configurations displayed behavior consistent with Reynolds number invariance as Re_D was increased. The change in C_p with increasing Re_D was initially rapid, particularly in the 1 to 3 million range. This dependence tapers off at sufficiently high Reynolds numbers, with power coefficient values above approximately 4×10^6 remaining essentially constant.

3.2. High Reynolds Number Limit

Prior work in [2] indicated that the correct parameter to characterize invariance was the chord-based Reynolds number defined as:

$$Re_c = \frac{\rho c(U + \omega R)}{\mu} = Re_D \frac{c}{D}(1 + \lambda) \tag{4}$$

The definition of Re_c combines the effects of λ and Re_D into one non-dimensional parameter. The choice of velocity scale is convenient because the current setup does not permit measurements of the velocity at the rotor blade itself. Instead, the value of Re_c represents the maximum possible chord-based Reynolds number a blade could encounter at a particular operating point (i.e. for a given λ and Re_D value). This parameter has no direct dependence on the solidity. The effect of changing σ by altering the number of blades can enter indirectly via λ as a change in the global performance of the rotor (as seen in figure 2).

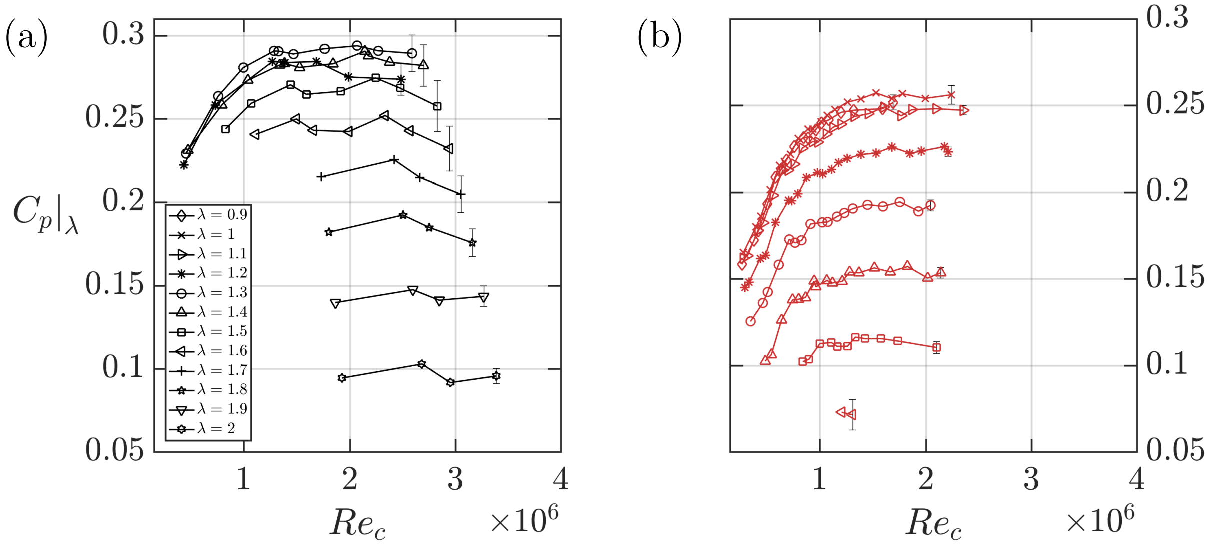

Figure 3. Power coefficient trends with tip speed ratio and chord Reynolds number. The plots correspond to the two different rotors, (a) 3 bladed, and (b) 5 bladed. Legend applies to both plots. Note that not all tip speeds are available for both rotor configurations, only $\lambda \in [1.2, 1.6]$ overlap. Error bars show maximum uncertainty associated with each curve.

Figure 3 shows the interpolated power coefficient as a function of the blade Reynolds number for a given tip speed ratio. Interpolation of the data was necessary since the model did not operate at fixed values of λ, instead operating points were specified as brake loads which give a wide range of tip speeds for each power curve. In these plots the λ location of peak power coefficient can be readily traced. For the 3 blade unit, $\lambda = 1.3$ corresponds to the peak in C_p except at the lowest tested Reynolds numbers where overlap is seen with the neighboring λ values (1.2 and 1.4). The 5 blade unit appears to maintain the peak in C_p at $\lambda = 1$ for all Reynolds numbers, although the neighboring tip speed values are quite close for low Re_c.

Interestingly, the 3 blade data shows a plateau in C_p above $Re_c \approx 1.5 \times 10^6$ which, as noted previously, was found to characterize trends observed in the 5 blade data. In particular, for $\lambda \in [1.2, 1.5]$ asymptotic behavior is clearly evident. The values of λ outside this range also exhibit relatively constant behavior for the given range of Re_c. The trends at low Re_c cannot be confirmed for all tip speeds because some points lie outside the operating envelope of the 3 blade turbine (in particular $\lambda = 1.7$ to 2.0). An additional observation regarding some of the higher tip speed cases, in particular the $\lambda = 1.6$ case, a downward trend in C_p appears at high Re_c. This is most likely due to the increased experimental uncertainty in the power coefficient at high tip speed ratios. At these operating points, the measured torque is small since the rotational speed makes up a considerable portion of the shaft power. This is denoted by the error bars at each tip speed ratio which represents the maximum error associated with each curve. The 5 blade data more clearly demonstrates asymptotic behavior simply due to the relatively small measurement error.

The plots of figure 3 have shown that for a given value of λ, turbine performance will not depend on the blade Reynolds number when $Re_c \geq 1.5 \times 10^6$, for both 3 and 5 blade

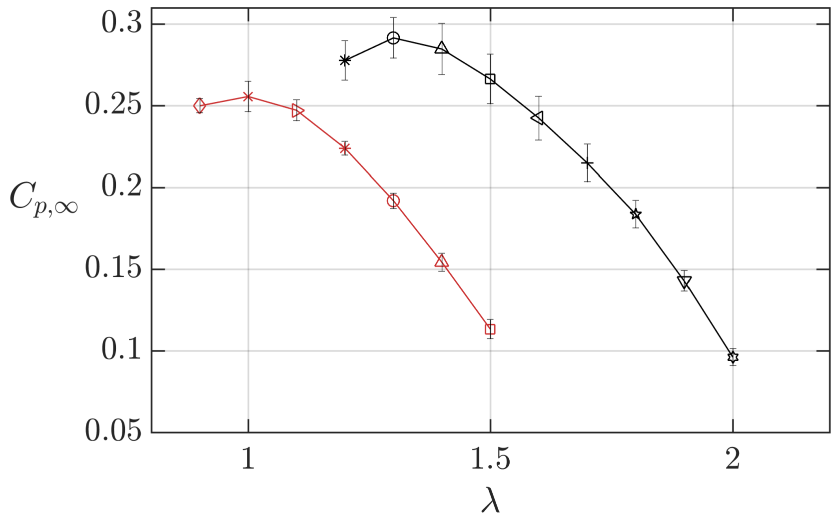

Figure 4. Reynolds invariant power coefficient as a function of tip speed ratio. Symbol color corresponds to the blade number with 3 blade data in black and 5 blade in red. Symbol shapes correspond to λ, as in figure 3.

configurations. This indicates that despite different performance curves for the two, the underlying flow physics scale with Reynolds number in a similar manner. To further investigate this claim, the Reynolds number invariant power coefficient, $C_{p,\infty}$ is defined. This parameter is the average C_p taken for data above a cutoff Reynolds number, $Re_c \geq 1.5 \times 10^6$, evaluated at specific λ values. With this information, the invariant power curve can be reconstructed and is shown in figure 4. Experiments and numerical simulations of either the 3 or 5 blade geometry will return either the black or red curve, respectively, if the blade Reynolds number is above $Re_c \geq 1.5 \times 10^6$.

The near asymptotic behavior of C_p is also of interest since many experiments and simulations operate at reduced Reynolds numbers. The data of figure 3 has been normalized with the $C_{p,\infty}$ curves of figure 4. In this way, the values will approach unity as Re_c is increased, as shown in figure 5 for the 3 and 5 blade turbine geometries. Both turbines appear to collapse on the same curve, indicating that although the solidity changes the actual value of $C_{p,\infty}$, it does not influence the relative change in the power coefficient as it approaches the asymptotic value. Some scatter is present for the 3 blade data, especially at higher Re_c values, but this was expected due to the additional uncertainty in these measurements as discussed earlier. Thus the single non-dimensional value of Re_c characterizes the rate at which the invariant power coefficient is achieved irrespective of the individual λ or Re_D chosen. Furthermore, no direct dependence is observed when changing the solidity, at least for the two turbine geometries studied.

4. Conclusions

The study of two different turbine solidities has led to further insight into the behavior of VAWTs at low Reynolds numbers and the transition to high Reynolds number behavior. In accordance with prior work, the power coefficient was seen to increase with Reynolds number. Furthermore, asymptotic behavior was observed above a cutoff Reynolds number of $Re_c \geq 1.5 \times 10^6$. This value of Re_c was found to characterize both solidities tested ($\sigma = 0.67$ for the 3 blade and $\sigma = 1.12$ for the 5 blade unit). The Reynolds invariant power curves were then found using this criteria. Finally, the rate at which the power coefficient approaches its asymptotic value appears

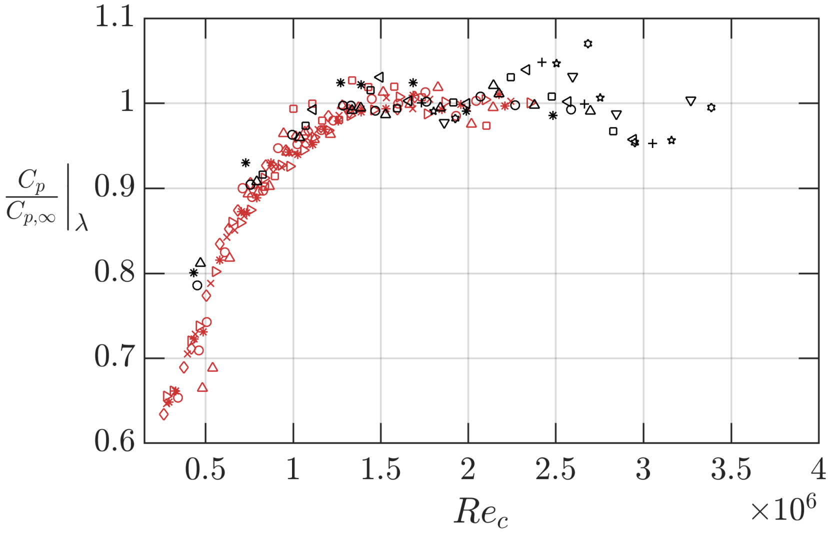

Figure 5. Power coefficient normalized with the value at Reynolds number invariance, for a given tip speed ratio. Symbol color corresponds to the blade number with 3 blade data in black and 5 blade in red. Symbol shapes correspond to λ, as in figure 3.

to depend only on Re_c and not directly on solidity, for the two cases tested in this work.

5. References

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